A6 Equations and graphs

The world is currently trying to respond to the effects on the environment of global warming. These effects are being predicted by the use of complex mathematical models. The models use mathematical functions along with current data on a wide range of variables.

What's the point?

The graphs of mathematical functions show how one quantity changes in relation to another. A mathematical model uses this idea to represent the real world by using a number of mathematical functions to represent a range of changing quantities and factors.

Orientation

What I need to know	What I will learn	What this leads to
A1 Plot graphs and find equations of straight lines **A5** Solve simultaneous equations	■ Draw and interpret real-life graphs ■ Solve equations by trial and improvement and graphically ■ Plot more complex graphs	**A-level** Maths, Sciences, Economics
N3 Use compound measures		Engineering

Check in

You should be able to

■ **evaluate formulae**

1 Evaluate these expressions for **i** $x = 3$ **ii** $x = -2$.

a x^2 **b** x^3 **c** $2x^2$ **d** $x^3 + x$

e $3x^2 - x$ **f** $2x^3 + 2x$ **g** $x^3 + 4x^2$ **h** $2x^3 - 4x^2 - x$

■ **use compound measures**

2 Give the speed in miles per hour for each journey.

a 20 miles is travelled in 1 hour. **b** 40 miles is travelled in $\frac{1}{2}$ hour.

c 30 miles is travelled in 20 minutes. **d** 120 miles is travelled in 5 hours.

■ **interpet straight line graphs**

3 For each line give its

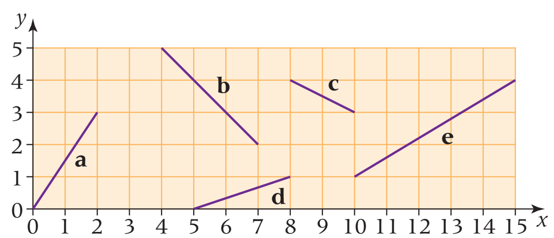
i gradient **ii** y-axis intercept **iii** direction.

a $y = 3x + 4$ **b** $y = 10 - 4x$ **c** $2y = 8x + 10$

d $2y - 4x = 15$ **e** $y = 7$ **f** $x = 2y - 4$

■ **find the equation of a straight line**

4 Give the gradient of each line segment.



Rich task

How bouncy is a ball?
Find ways to compare the bounciness of different balls used in sports such as football, tennis, golf, squash, table tennis, *etc.*
Investigate and write a report on your results.

Distance–time graphs

This spread will show you how to:
- Draw and interpret distance–time graphs
- Understand and use compound measures, including speed

Keywords
Distance
Speed
Time

- You can represent a journey on a distance–time graph.
- **Time** is always plotted on the horizontal axis.
- **Distance** is plotted on the vertical axis.

This graph shows Dan's journey on his bike.

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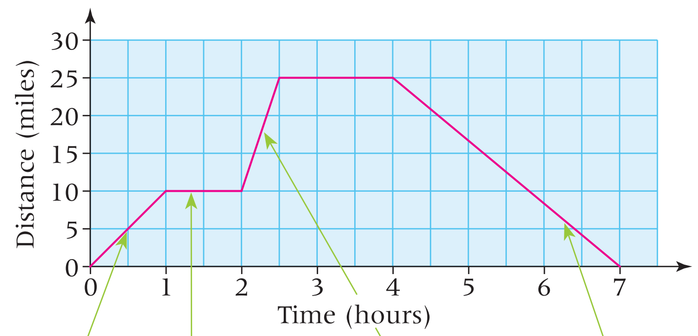


Dan covers 10 miles in 1 hour. His **speed** is 10 mph.

Dan's distance does not change. He has stopped for 1 hour.

Dan covers 15 miles in $\frac{1}{2}$ hr. His speed is 30 mph. He cycles much faster than before (maybe downhill).

Dan returns home. He covers 25 miles in 3 hours.

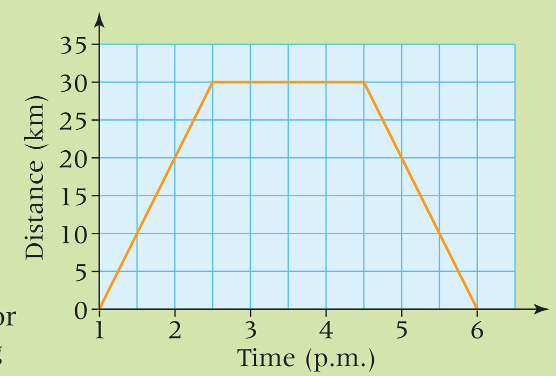
Example

Janine leaves home at 1 p.m. and cycles to her friend's house, 30 km away, at a speed of 20 km/h. She stays for 2 hours, then cycles home, arriving at 6 o'clock.

Draw a distance-time graph to represent the journey and determine her speed on the way home and her average speed for the entire journey.

On the journey home, Janine covers the 30 km in $1\frac{1}{2}$ hours. This means that she has covered 10 km in each half hour and, hence, 20 km in each hour. Her speed is 20 km/h.

Since she covers 60 km in 5 hours, her average speed for the whole journey, including the stop, is 12 km/h.



Janine returning home, is shown by the graph going down, back to the x-axis.

average speed = $\frac{\text{total distance}}{\text{total time}}$

Exercise A6.1

Grade D/C

1 The distance–time graph shows the journey of a car between Birmingham and Stoke-on-Trent.

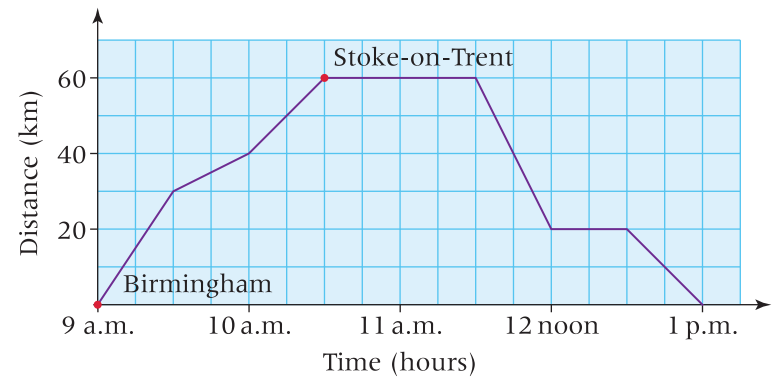
a How far is it from Birmingham to Stoke-on-Trent?

b For how long did the car stop?

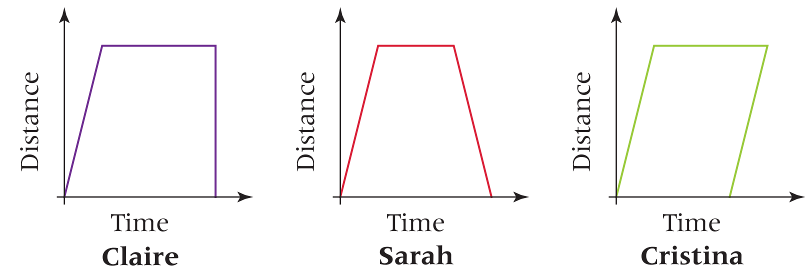
c What was the speed of the car for the first part of the journey?

d Between which two times was the car travelling fastest?

e What was the average speed of the car for the whole journey?



2 Three students have drawn distance–time graphs. Two have made mistakes. Which two students have made a mistake and what mistake is it?



3 Construct a distance–time graph to show each journey.

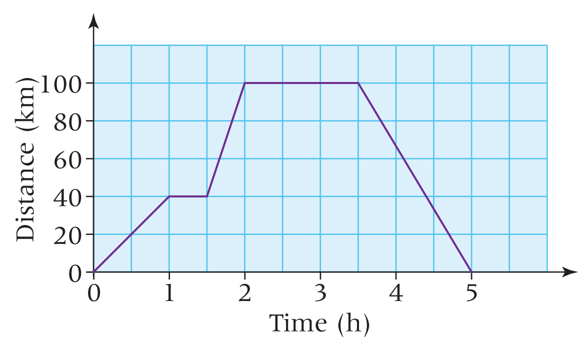
a A car travels between Bristol and London. On the outward journey, it travels the 120 miles to London in $2\frac{1}{4}$ hours. The driver remains in London for $1\frac{1}{2}$ hours. The car travels half way back to Bristol at 40 miles per hour, as the motorway is busy, then the remaining distance at 80 miles per hour.

b Two brothers both went to see each other on the same day. Henry left his home at 2 p.m. to go and see Leo, who lives 5 miles away. Henry walked at an average speed of 4 miles per hour, but he stopped half way for a 15 minute rest. At 2:30 p.m., Leo set out on his bicycle from his home in order to go and visit Henry. He cycled straight there in $\frac{1}{2}$ hour.

Draw the two journeys on one graph.

A03 Problem

4 The following graph represents the journey of a car. Construct a graph of speed (km/h) against time (h) for this journey.



A6 Summary

Check out

You should now be able to:

- Use systematic trial and improvement to find approximate solutions to equations
- Generate points and plot graphs of simple quadratic, simple cubic and reciprocal functions
- Find approximate solutions of equations from their graphs, including one linear and one quadratic
- Draw and interpret distance-time graphs
- Draw and interpret non-linear graphs modelling real-life situations
- Construct straight line and quadratic graphs from real-life problems

Worked exam question

a Complete the table of values for $y = 2x^2 - 4x$

x	-2	-1	0	1	2	3
y	16		0			6

(2)

b On the grid, draw the graph of $y = 2x^2 - 4x$ for values of x from -2 to 3 (2)

c **i** On the same axes, draw the straight line $y = 2.5$
ii Write down the values of x for which $2x^2 - 4x = 2.5$ (2)

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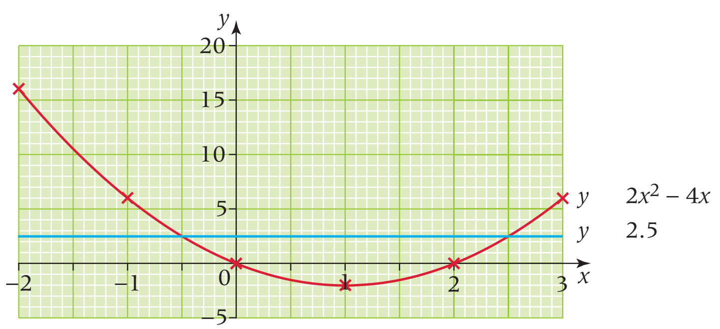
a

When $x = 2$ $\quad y = 2 \times 2^2 - 4 \times 2 = 0$
When $x = 1$ $\quad y = 2 \times 1^2 - 4 \times 1 = -2$
When $x = -1$ $\quad y = 2 \times (-1)^2 - 4 \times -1 = 6$

x	-2	-1	0	1	2	3
y	16	6	0	-2	0	6

b



Draw a smooth curve using the points from part **a**.

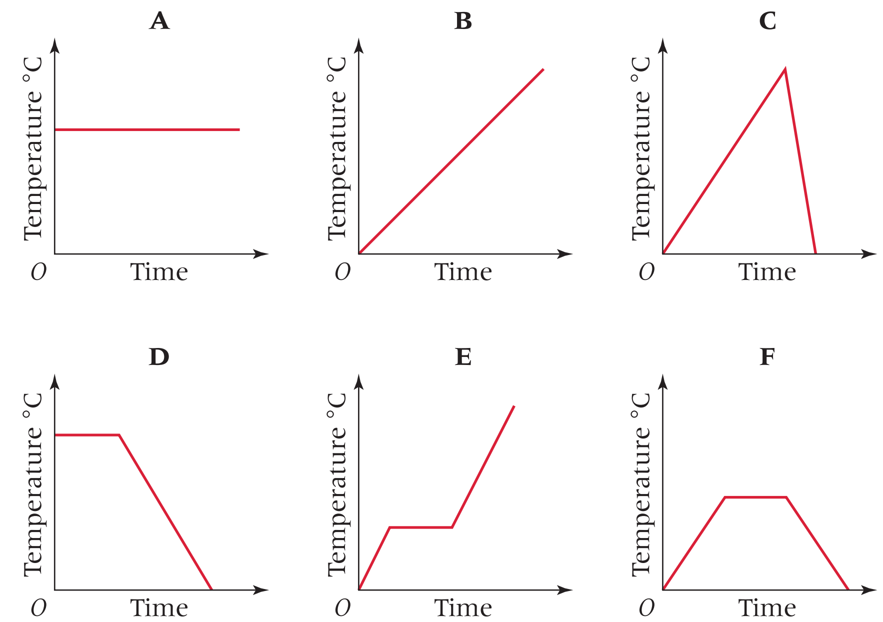
c **i** See above **ii** -0.5 and $x = 2.5$

The question asks for values of x (i.e. more than one)

Exam questions

1 The equation $x^3 + 5x = 20$
has a solution between 2 and 3
Use a trial and improvement method to find this solution.
Give your answer correct to one decimal place.
You must show ALL your working. (4)

2 Here are six temperature graphs.



Each sentence in the table describes one of the graphs.
Write the letter of the correct graph next to each sentence.
The first one has been done for you.

The temperature starts at 0°C and keeps rising.	**B**
The temperature stays the same for a time then falls.	
The temperature rises and then falls quickly.	
The temperature is always the same.	
The temperature rises, stays the same for a time and then falls.	
The temperature rises, stays the same for a time and then rises again.	

(3)
(Edexcel Limited 2008)